# RAMAKRISHNA MISSION VIDYAMANDIRA

**(Residential Autonomous College under University of Calcutta)**

**B.A./B.SC. SECOND SEMESTER EXAMINATION, MAY-JUNE 2013**

**FIRST YEAR**

Date : 24/05/2013 **Mathematics (General)**

Time : 11am – 2pm Paper : II Full Marks : 75

**[Use separate answer books for each group]**

## Group-A

Answer **any three** questions: 3x5=15

1. Find the nature of the following equation and reduce it to it's canonical form:
$$8x^2 - 12xy + 17y^2 - 4x - 22y + 13 = 0$$ 5

2. a) The co-ordinate axes are rotated through an angle of $\frac{\pi}{3}$. If the transformed co-ordinates of a point are $\left(2\sqrt{3}, -6\right)$, find it's original co-ordinates. 1

   b) If the polar of a point $(\alpha, \beta)$ with respect to the parabola $y^2 = 4ax$ touches the parabola $x^2 = 4by$ then show that the locus of the point $(\alpha, \beta)$ is $xy + 2ab = 0$. 4

3. If the straight lines joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and $2x + 3y + k = 0$ be at right angles, then show that, $6k^2 + 5k + 52 = 0$. 5

4. Find the locus of the points of intersection of pair of perpendicular tangents of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 5

5. PSP$'$ is a focal chord of a conic. Prove that the angle between the tangents at p and p$'$ is $\tan^{-1}\left(\frac{2e\sin\alpha}{1-e^2}\right)$, where $\alpha$ is the angle between the chord and the major axis. 5

## Group-B

Answer **any three** questions: 3x5=15

6. Prove that the medians of a triangle meet at a point which divides each median in the ration 2:1. 5

7. a) Determine the values of $\lambda$ and $\mu$ for which the vectors $\left(-3\vec{i} + 4\vec{j} + \lambda\vec{k}\right)$ and $\left(\mu\vec{i} + 8\vec{j} + 6\vec{k}\right)$ are collinear. 2

   b) Show that the four points $\vec{\alpha} = 6\vec{a} - 4\vec{b} + 10\vec{c}$, $\vec{\beta} = -5\vec{a} + 3\vec{b} - 10\vec{c}$, $\vec{\gamma} = 4\vec{a} - 6\vec{b} - 10\vec{c}$, $\vec{\delta} = 2\vec{b} + 10\vec{c}$ are coplanar. 3

8. a) If $\left|\vec{\alpha} + \vec{\beta}\right| = \left|\vec{\alpha} - \vec{\beta}\right|$, prove that $\vec{\alpha}$ is perpendicular to $\vec{\beta}$. 2

   b) Show by vector method that the angle in a semi-circle is a right angle. 3

9. Find the vector equation of the plane perpendicular to the vector $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$ and passing through the terminal point of the vector $\vec{b} = \vec{i} + 5\vec{j} + 3\vec{k}$. 5

10. Find by vector method the volume of the tetrahedron whose vertices are, A(2, -1, -3), B(4, 1, 3), C(3, 2, -1), D(1, 4, 2). 5

## Group-C

Answer **any five** questions: 5X5=25

11. a) Show that $\left\{\frac{3n+1}{n+2}\right\}$ is a bounded sequence.

    b) Let, $S_n = \frac{n+1}{2n+1}$, verify whether $\{S_n\}$ is a monotone decreasing sequence? 3+2

12. a) If $\sum_{n=1}^{\infty} a_n$ be a convergent series of real numbers, prove that $a_n \to 0$ as $n \to \infty$.

b) Test the convergence of the series: $1+\frac{1}{2}\cdot\frac{1}{3}+\frac{1.3}{2.4}\cdot\frac{1}{5}+\frac{1.3.5}{2.4.6}\cdot\frac{1}{7}+\cdots$ 2+3

13. State and prove Lagrange's Mean Value Theorem. 1+4

14.a) Find the values of a and b such that $\lim_{x\to 0}\frac{ae^x+be^{-x}+2\sin x}{\sin x+x\cos x}=2$ 3

b) Find the extreme values of the function f in its domain, where $f(x)=\frac{\log x}{x}$ . 2

15. Find all the asymptotes of $x^3-2x^2y+xy^2+x^2-xy+2=0$. 5

16. Find the envelope of the family of ellipse $\frac{x^2}{a^2}+\frac{x^2}{b^2}=1$, where the parameters a, b are connected by a+b=k. 5

17. Use Maclaurin's infinite series to prove that for all real *x*, $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$ 5

18. a) Show that (1,0) is a node of the curve $x^4-2y^3-3y^2-2x^2+1=0$. 2

b) Using Lagrange's method of undetermined multiplier, find the extreme values of $f(x,y)=xy$ subject to the condition $5x+y=13$. 3

## Group-D

19. Answer **any one** question: 1×2

a) Evaluate $\int\frac{dx}{4+5\cos x}$. 2

b) Find the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x\cos x}{2\sin^2 x+5\cos^2 x}dx$. 2

20. Answer **any two** questions: 2×4

a) Evaluate $\int\frac{2\cos x+3\sin x}{3\cos x+2\sin x}dx$. 4

b) Find reduction formula for $\int\tan^n x\,dx$, n being a positive integer greater than 1. Hence find $\int_0^{\frac{\pi}{4}}\tan^4 x\,dx$. 4

c) Evaluate $\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\cdots+\frac{1}{n}\right]$. 4

## Group-E

21. Answer **any one** question: 1×2

a) Find the order and the degree of the differential equation $\frac{d^3y}{dx^3}+y=\sqrt{1+\frac{dy}{dx}}$. 2

b) Construct a differential equation by the elimination of the constants a and b from $ax^2+by^2=1$. 2

22. Answer **any two** questions: 2×4

a) Solve: $x^2dy+y(x+y)dx=0$. 4

b) Solve: $(x^2y^3+2xy)dy=dx$. 4

c) Find the general and singular solution of $y=px+\sqrt{a^2p^2+b^2}$, $p=\frac{d}{dx}$. 4